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Final Report for Period: 06/2002 - 05/2006
Submitted on: 07/15/2006
Principal Investigator: Harrell, Evans M.
Award ID: 0204059
Organization: GA Tech Res Corp - GIT
Title:
Spectra, Geometry, and Asymptotics of Some Differential Equations of Mathematical Physics

Project Participants

Senior Personnel

Name: Harrell, Evans
Worked for more than 160 Hours: Yes
Contribution to Project:

Post-doc

Graduate Student

Undergraduate Student

Technician, Programmer

Other Participant

Research Experience for Undergraduates

Organizational Partners

Other Collaborators or Contacts

Harrell's active collaborations in 2004 included:
1. Spectral analysis of nonlinear PDEs containing the p-Laplacian,
with J. Fleckinger and F. de Thelin of Toulouse. (A manuscript has been submitted.)
2. Universal bounds and Reilly-type inequaliteis for eigenvalues of Schroedinger operators on manifolds, with A. El Soufi and S. Ilias of Tours. (A manuscript is in preparation.
3. Optimal shapes of charged nanoscale devices, with P. Exner of Prague and M. Loss of Georgia Tech.

Activities and Findings

Research and Education Activities:
Research. Harrell made progress on

1. Spectral theory of Schroedinger operators on surfaces.

2. Estimates of the eigenvalue gaps of the p-Laplacian and similar non-linear PDEs.

3. Lower bounds for the fundamental eigenvalue of elliptic, potentially non-linear operators with Neumann boundary conditions on part of the boundary.

4. Theory of quantum resonances

Harrell presented his research in numerous seminars and meetings.

Two graduate students, Mr. Edward White and Ms. Selma Yildirim, began thesis work with Harrell, which has not yet led to publishable results.

Although Harrell was heavily involved in education, both as a teacher and as Associate Chair for Graduate Studies and Research, this was not specifically part of this sponsored project.

Findings:

1. Harrell showed that the fundamental eigenvalues of a family of elliptic, generally nonlinear operators satisfy Hardy-type bounds from below. This is reported in J. Comput. Appl. Math. 194(2006)26-35.

2. In Letters in Math. Phys.75(2006)225-233, Exner, Harrell, and Loss investigated problems in electrostatics and in quantum mechanics in the presence of a uniformly charged loop, and showed that in some Circumstances the configuration that optimizes energies is circular. This involved proving a new isoperimetric theorem of a classical geometric type.

3. An article on sum-rule identities for partial differential operators on hypersurfaces will appear in Communications in Part. Diff. Eq.

4. Together with Fleckinger and de Thelin, Harrell proved the first quantitative estimates on the second eigenvalue of the p-Laplacian on a domain. (To appear in Bulletin des Sciences MathÚmatiques.)

5. Harrell wrote an invited review article on the mathematical theory of atomic resonances, which will appear in the Proceedings of Symposia in Pure Mathematics.

6. El Soufi, Harrell, and Ilias have proved a significant extension of Reilly's inequality of spectral geometry, showing that for the Laplacian and many Schr\'odinger operators on submanifolds, every eigenvalue can be bounded in terms of the mean curvature. A manuscript is in preparation.

Training and Development:

At the end of 2004, Harrell started working with a new PhD student, Selma Yildirim, who disposed of her qualifying exams and other non-thesis requirements in 2006. She is currently seeking a specific topic in the spectral theory of Schr\'odinger operators.

In addition, in 2006 Mr. Edward White has begun work on a master's thesis on extremal problems of convex geometry. Mr. White is an African American.

Outreach Activities:

Harrell spoke to a number of college Math Clubs, and in October, 2004, participated in the National 'MathFest' of the National Association of Mathematicians, an event aimed at Minority mathematics students.

As Associate Dean of Sciences, Harrell has participated in a grant proposal and has obtained commitments from Georgia Tech to partner with local high schools in offering college-level mathematics to talented pre-college students.

Journal Publications

Q. Chen, K. A. Bowman, E.M. Harrell, and J.D. Meindl, "Double Jeopardy in the nanoscale Court?", IEE Circuit and Devices, p. 28, vol. 19, (2003). Published

Q. Chen, E.M. Harrell, and J.D. Meindl, "A Physical Short-Channel Threshold Voltage Model for Undoped Symmetric Double-Gate MOSFET's", IEEE Transactions on Electron Devices, p. 1631, vol. 50, (2003). Published

E.M. Harrell, "Commutators, eigenvalue gaps, and mean curvature in the theory of Schr?dinger operators", Journal of Differential Geometry, p. , vol. , ( ). Submitted

Books or Other One-time Publications

Web/Internet Site

Other Specific Products

Contributions

Contributions within Discipline:

The work on spectra of Schroedinger operators on surfaces and hypersurfaces advances mathemaics through its novel connections among algebra, geometry,a nd analysis. It advanced mathematical physics because it rigorously elucidates the possible energy levels of a quantum particle on a thin structure such as a curved nanoscale waveguide.

Other parts of the project advance the understanding of nonlinear elliptic PDEs.

Contributions to Other Disciplines:

Harrell's work on thin structures has implications for the electrical properties of quantum wires, waveguides, and other nanoscale devices.

Contributions to Human Resource Development:

Harrell is beginning to guide a female doctoral student in mathematics, Selma Yildirim.

Contributions to Resources for Research and Education:

Contributions Beyond Science and Engineering:

Categories for which nothing is reported:

Organizational Partners

Any Book

Any Web/Internet Site

Any Product

Contributions: To Any Resources for Research and Education

Contributions: To Any Beyond Science and Engineering